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SALIENT FEATURES OF INERTIAL STRETCHING OF A HIGH-GRADIENT CONDUCTING ROD IN A LONGITUDINAL LOW-FREQUENCY MAGNETIC FIELD

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From the viewpoint of the model of a uniformly stretching cylindrical incompressible rigid-plastic conducting rod, the salient features of deformation of metal cumulative jets in a longitudinal low-frequency magnetic field are considered. The electromagnetic processes proceeding in the jet segments when they enter or leave the region with a magnetic field have been investigated. It is shown that the electromagnetic forces arising as the result of deformation of the jet with the magnetic field that has penetrated into its material should inhibit the development of plastic instability, extending the stretching stage of the jet before it breaks up into separate segments and thus creating prerequisites for increasing its penetrating power capacity. The magnetic-field parameters that enable one to expect a noticeable manifestation of the stabilizing effect have been estimated.

Explosion of an axially symmetrical charge of an explosive with a groove having a thin metal coating may lead to the formation of high-velocity cumulative jets (CJ) which, when in free flight, first stretch, remaining solid, and then break up into separate nondeformable segments [1]. The cause of the breakup of high-gradient CJs from such materials as copper, aluminum, and niobium is the plastic instability, which is realized as the appearance and development of numerous necks on the jet [2–4]. In this case, the necking stage is preceded by the so-called inertial stage of jet stretching at which its segments stretch uniformly, remaining almost cylindrical in shape [4, 5]. The efficiency of the penetrating action of a CJ is determined by the length of the segments formed as the result of its breakup.

The development of plastic instability on a CJ can be hindered and its effective length can thereby be increased if the stretching jet is exposed to a longitudinal low-frequency magnetic field [6]. The experimental results presented in [7] point to a certain positive effect of such a magnetic field on the efficiency of the penetrating action of the CJ. The stabilizing effect of the magnetic field on the process of CJ stretching is associated with the appearance of additional electromagnetic forces in the jet deforming in the magnetic field that hinder the development of necks.

In the present paper, we consider from theoretical positions the salient features of CJ deformation in a longitudinal low-frequency magnetic field and estimate the parameters of its action at which a pronounced manifestation of the stabilizing effect can be expected.

By low-frequency action is meant the action of a magnetic field the character of whose change with time and space provides the possibility of its complete diffusion into the CJ material, so that for a considerable part of the time of their deformation the CJ segments are stretching with the magnetic field that has penetrated into their depth. In practice, such an action on a CJ is realized with the aid of an elongated solenoid located ahead of a shaped charge on the path of the CJ motion. With the proper choice of parameters

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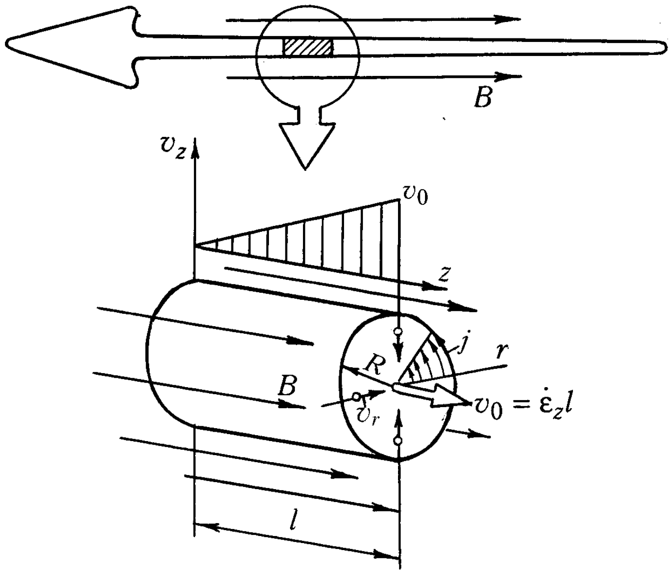


Fig. 1. Calculated scheme of cumulative-jet-segment deformation in a magnetic field.

of the electric circuit of the solenoid, we can reach a point at which the CJ fragments passing through it will, for some time, depending on their velocity and the solenoid length, stay in the magnetic field with a practically constant intensity. Taking this into account, in order to simplify the analysis of the CJ behavior in a magnetic field, we restrict ourselves to the consideration of the case of constant fields.

Model of Magnetic-Field Evolution in a Deforming CJ. In investigating the influence of a magnetic field on the stress-strain evolution in the jet, its segments were regarded as portions of a cylindrical incompressible rigid-plastic conducting rod stretching with the axial rate of strain $\dot{\varepsilon}_z$. Within the framework of such an approximation, the problem is actually reduced to the investigation of the deformation of individual plane sections of the CJ in each of which the jet particle motion occurs only in the radial direction, the only component of the magnetic induction vector B is directed along the jet axis, and the induced eddy currents arising in its material are azimuthal (Fig. 1).

Assuming a weak manifestation of the skin-effects, we will presume that the space density of induced currents j is linearly distributed along the radius r of the cross section of the rod at all times:

$$j(r, t) = \beta(t)\, r\,, \tag{1}$$

where β is a coefficient depending only on time.

Integrating relation (1) over the rod radius, for the radial distribution of the magnetic field induction in the jet material at all times, we obtain the dependence of the parabolic form

$$B(r, t) = B_e + \frac{\mu_0 \beta}{2}(R^2 - r^2)\,, \tag{2}$$

where R is the outer radius of a jet segment; $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the permeability of vacuum.

To determine the coefficient β in the distribution of induced currents (1), we make use of the law of electromagnetic induction [8], which, in our case of one-dimensional axially symmetrical motion of an incompressible conducting medium (see Fig. 1), takes on the following form:

$$\frac{dB}{dt} = \dot{\varepsilon}_z B - \frac{1}{r}\frac{\partial (Er)}{\partial r}\,,$$

where E is the azimuthal electric-field strength.

Writing this equation for the particles of the stretching rod located on its axis ($\tau = 0$), taking into account Ohm's law in the differential form $E = \eta j$ (η is the specific resistance of the jet material) as well as taking into account the radial distributions of the space density of currents (1) and magnetic induction (2), we obtain the ordinary differential equation with respect to the coefficient β

$$\frac{d\beta}{dt} + 2\beta\left(\frac{2\eta}{\mu_0 R^2} - \dot{\varepsilon}_z\right) = \frac{2}{\mu_0 R^2}\left(\dot{\varepsilon}_z B_e - \frac{dB_e}{dt}\right), \tag{3}$$

which, together with (2), makes it possible to describe the magnetic-field evolution in the jet material. The outer radius R of the jet segment entering into relations (2) and (3), with the use of the law of conservation of mass for an incompressible medium, was assumed to be decreasing in the course of time in accordance with the differential dependence $dR/dt = -\dot{\varepsilon}_z R/2$.

To substantiate the simplified approach taken for describing the electromagnetic processes in the rod, we consider the problem on the penetration of a longitudinal constant magnetic field with induction $B_e = B_0$ into a nondeformable ($\dot{\varepsilon}_z = 0$) conducting cylindrical rod with an invariable radius $R = R_{in}$. The exact analytical solution of this diffusion problem obtained by methods of mathematical physics [9] gives the following series for calculating the magnetic-field induction in the rod material:

$$B(r, t) = B_0\left\{1 - 2\sum_{k=1}^{\infty}\frac{J_0(\nu_k r/R_{in})}{\nu_k J_1(\nu_k)}\exp\left(-\frac{\nu_k^2\eta}{\mu_0 R_{in}^2}t\right)\right\}.$$

With the use of the simplified approach the solution of the same problem reduces to the integration of the equation below following from (3)

$$\frac{d\beta}{dt} + \frac{4\eta}{\mu_0 R_{in}^2}\beta = 0$$

under the initial conditions $\beta(0) = -2B_0/(\mu_0 R_{in}^2)$ providing the absence of a magnetic field on the axis of the rod at zero time. As a result, for the coefficient β and the magnetic induction distribution in the rod determined on its basis from (2), we obtain the expressions

$$\beta = -\frac{2B_0}{\mu_0 R_{in}^2}\exp(-t/\tau), \quad B(r, t) = B_0\left[1 - \exp(-t/\tau)\left(1 - \frac{r^2}{R_{in}^2}\right)\right],$$

where τ is the time constant of magnetic-field diffusion, which has the form

$$\tau = \frac{\mu_0 R_{in}^2}{4\eta}. \tag{4}$$

As seen from the change in the magnetic induction B_{rod} on the axis ($r = 0$) of the nondeformable rod placed in the magnetic field, illustrated in Fig. 2, there is a close agreement between the exact and approximate solutions, which enables us to regard the approach accepted for calculating the magnetic-field evolution in the jet to be reasonable.

Unlike the nondeformable rod, the change in the magnetic field in a stretching CJ, besides the diffusion factor, should also be associated with the factor of the generation of a field in the jet, which, in accordance with the effect of "freezing-in" of a magnetic field into a substance [8], is due to the elongation of the

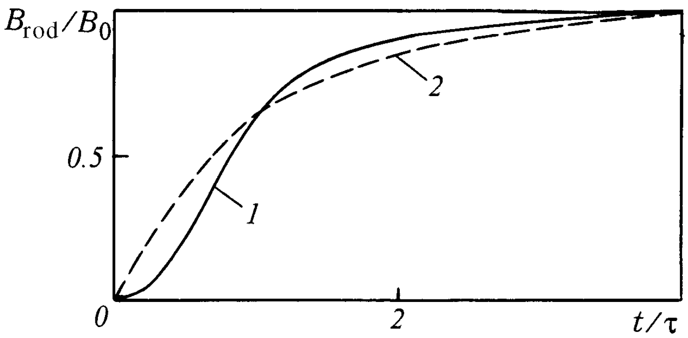


Fig. 2. Change in the magnetic induction on the axis of a nondeformable cylindrical rod placed in a magnetic field: 1) exact analytical solution; 2) approximate solution.

CJ particles in the direction of the magnetic induction lines of the field penetrating into them. It is noteworthy that at the stage of penetration of the external field into the jet both these factors lead to the strengthening of the field in the jet. But if as a result of the generation the field intensity in it exceeds the intensity of the external field, the diffusion processes, on the contrary, should weaken further "pumping" of the field in the jet.

Turning to the elucidation of the salient features of deformation of CJ segments in a magnetic field, we consider the case of uniform stretching of the jet, which, as mentioned above, corresponds to the inertial stage of its stretching. We shall thereby assume the intensity of the acting field to be constant: $B_e = B_0$.

If at the time of onset of the action ($\tau = 0$) the jet segment deforms with the axial rate of strain $\dot{\varepsilon}_{\rm zin}$ and has radius $R_{\rm in}$, then with its further uniform stretching these parameters will change in accordance with the dependences [5]

$$\dot{\varepsilon}_z = \frac{\dot{\varepsilon}_{\rm zin}}{1 + \dot{\varepsilon}_{\rm zin} t}, \quad R = \frac{R_{\rm in}}{\sqrt{1 + \dot{\varepsilon}_{\rm zin} t}} .$$

With these relations taken into account, Eq. (3), describing the electromagnetic processes in the jet, for the inertial stage of deformation of its segments, can be given in the following dimensionless form:

$$\frac{d\overline{\beta}}{d\overline{t}} + 2\overline{\beta}\left(\frac{1 + \kappa\overline{t}}{2} - \frac{\kappa}{1 + \kappa\overline{t}}\right) = 2\kappa , \tag{5}$$

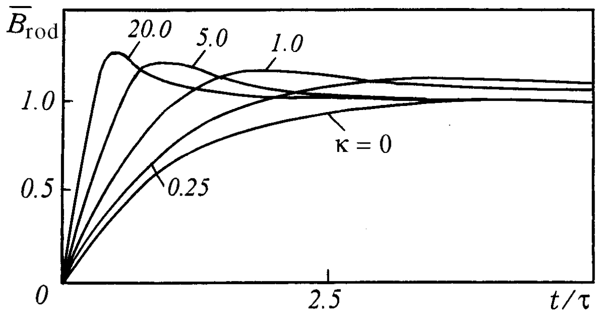
where the dimensionless distribution coefficient of induced currents $\overline{\beta}$ and time $\overline{t}$ are determined by the relations $\overline{\beta} = \beta\mu_0 R_{\rm in}^2/B_0$ and $\overline{t} = t/\tau$, and the dimensionless parameter κ is of the form

$$\kappa = \dot{\varepsilon}_{\rm zin}\tau . \tag{6}$$

The diffusion time constant τ entering into the expression for the dimensionless time $\overline{t}$ and parameter κ is determined by the radius of the jet segment $R_{\rm in}$ at the moment of onset of the action and, as before, is given by relation (4).

Analyzing the magnetic-field evolution in the jet segment, as the main characteristic of this process, we take the change in magnetic induction $B_{\rm rod}$ on the CJ-segment axis. The dimensionless quantity of this induction $\overline{B}_{\rm rod} = B_{\rm rod}/B_0$ in uniform stretching of the jet segment with the use of (2) is written in the form $\overline{B}_{\rm rod} = 1 + 0.5\overline{\beta}/(1 + \kappa\overline{t})$.

As follows from Eq. (5), in uniform deformation of the CJ the magnetic evolution in it is determined by the value of the dimensionless parameter κ, which, according to dependence (6), has the meaning of the



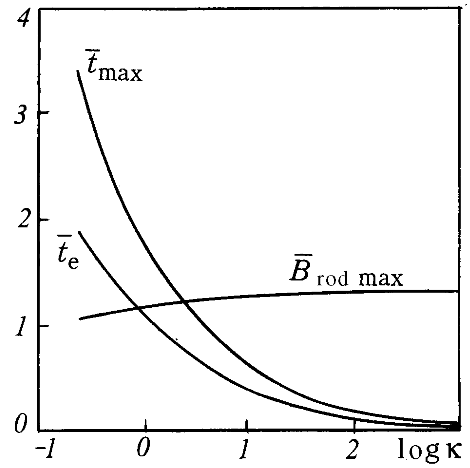


Fig. 3. Change in the magnetic field on the axis of a uniformly stretching segment of a cumulative jet when it enters the region of action.

Fig. 4. Influence of the dimensionless parameter κ on the characteristics of the magnetic-field evolution in a cumulative jet segment when it enters the region of action.

change in the relative length of the jet segment in the characteristic diffusion time τ. The time scale of jet strain and, consequently, the time of the field strengthening caused by it in the jet material is determined by the quantity $1/\dot{\varepsilon}_{zin}$. Taking this into account, we can easily establish that the parameter κ characterizes the ratio between the diffusion and field generation rates in the jet.

Electromagnetic Processes Proceeding when the CJ Segment Enters the Magnetic Field. To the entrance of a freely deforming CJ segment into the region with a constant magnetic field $B_e = B_0$ there corresponds the solution of Eq. (5) with the initial condition $\bar{\beta}(0) = -2$. The expression for determining the magnetic induction on the segment axis obtained thereby

$$\bar{B}_{rod} = 1 - (1 + \kappa\bar{t})\exp\left[-\bar{t}\left(1 + \frac{\kappa}{2}\bar{t}\right)\right]\left\{1 - \kappa\int_0^{\bar{t}} \frac{\exp\left[\bar{t}\left(1 + \frac{\kappa}{2}\bar{t}\right)\right]}{(1 + \kappa\bar{t})^2}\,d\bar{t}\right\}$$

includes an integral which is not expressed in the final analytical form.

The change in the magnetic induction on the axis of the jet segment getting into the magnetic field, established as the result of numerical calculations for various values of the parameter κ, is shown in Fig. 3. Owing to the stretching factor leading to a compression of the field penetrating into the jet, in time t_e the induction inside the jet becomes equal to the induction of the external field and then exceeds it. On reaching its maximum value of $B_{rod\ max}$ at time t_{max}, the magnetic-field intensity in the jet monotonically decreases, tending to the intensity of the external field. The dependences of the dimensionless quantities $\bar{t}_e = t_e/\tau$, $\bar{t}_{max} = t_{max}/\tau$, and $\bar{B}_{rod\ max} = B_{rod\ max}/B_0$ on the parameter κ obtained as the result of the calculations are illustrated in Fig. 4.

For the time t_e determining the moment of complete penetration of the magnetic field into the jet material, we obtained the following approximation:

$$t_e = \frac{1.05\tau}{\kappa^{0.42}}. \tag{7}$$

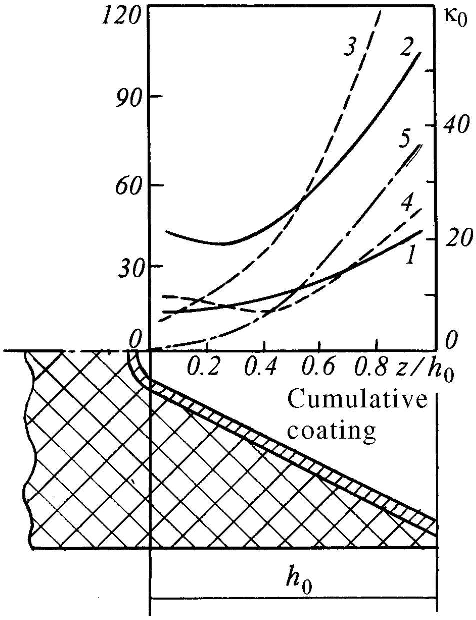


Fig. 5. Distribution of the parameters characterizing the processes of magnetic-field deformation and evolution by the cumulative jet segments formed from different parts of the coating, with the example of a laboratory shaped charge: 1) time of transition from the inertial to the necking stage of deformation; 2) breakup time; 3) time constant of magnetic-field diffusion τ_0; 4) magnetic-field penetration time t_{e0}; 5) dimensionless parameter κ_0.

The results obtained enable one to forecast the character of change in the magnetic field in the jet at the onset of action at different moments of its inertial stage of stretching. As the jet stretches uniformly, the main characteristics of the electromagnetic processes change, depending on the current stretch ratio n, in accordance with the relations

$$\tau = \frac{\tau_0}{n}; \quad \kappa = \frac{\kappa_0}{n^2}; \quad t_e = \frac{t_{e0}}{n^{0.16}},$$

where the values of τ_0, κ_0, and t_{e0} are calculated by the parameters of the jet segments at the instant they are formed and the stretch ratio is equal to the ratio of the current length of the segment to its initial length $n = l/l_0$. It is interesting to note that the decrease in the time of complete penetration of the field t_e when the onset of action is shifted to later stages of jet deformation is insignificant. This is due to the fact that, on the one hand, a decrease in the radius of the jet segments when they stretch leads to an increase in the diffusion rate of the magnetic field, but on the other hand, because of the simultaneous decrease in the axial rate of strain $\dot{\varepsilon}_z$ the role of the compression factor of the field decreases.

Figure 5 gives an idea about the actual ratio of the time of magnetic-field penetration into the CJ material to the time of the main stages of its deformation. In this figure, with the example of a laboratory shaped charge with a diameter d_z of 50 mm, which was used in experiments with passage of the CJ through a magnetic field described in [7], the time intervals (in microseconds) of different processes are compared. The shaped charge under consideration has a copper cumulative coating with a flare angle of 50°; the kinematic and geometric parameters of the CJ produced by this charge were determined on the basis of [10]. Figure 5 shows the distributions, over the jet segments formed from different parts of the coating, of the duration of the inertial stage of their stretching, the breakup time [4] as well as of the time constant of diffusion, the time of complete penetration of the magnetic field, and the parameter κ_0. All the characteristics of the electromagnetic processes relate to the moment of formation of CJ segments. From the same instant of time for each element the time of transition from the inertial to the necking stage of stretching and the breakup times are measured.

As is seen from Fig. 5, for the 50-mm shaped charge under consideration, complete penetration of the magnetic field into the CJ segments under the condition of the onset of its action the moment they are formed occurs exactly by the moment of transition from the inertial to the necking stage of stretching. Obviously, in a real situation, this time will be shifted to the onset of the necking stage because of the impossibility of placing the solenoid directly in the region of jet formation.

It is tempting to analyze the relationship between the time boundaries of the process of penetration of a magnetic field and the stages of CJ deformation for shaped charges of other sizes. As follows from relations (4), (6), and (7), the evolution of the field in the jet is associated with the manifestation of the scale factor. For geometrically similar jets from the same material with the similarity factor k_s, the penetration times of the magnetic field should be related as $k_s^{1.58}$. This means that with increasing diameter of the shaped charge the time of field penetration into the jet will be shifted to later stages of its strain. For example, for a shaped charge with a diameter d_z = 100 mm, this time, as estimates show, is shifted to approximately the middle of the necking stage of the CJ.

Electromagnetic Processes Proceeding when the CJ Segment Leaves the Magnetic Field. To simplify the analysis of the electromagnetic processes in CJ segments when they leave the region of action, we digress from the possible, by this time, inhomogeneity of strain of different portions of the jet caused by the development of plastic instability and rely, as before, on the model of a uniformly stretching cylindrical rod. Such a formulation of the problem appears to be not meaningless, since it permits one to elucidate the salient features of the proceeding processes at least at a qualitative level.

Equation (5) describing the magnetic field evolution for a uniformly deforming CJ segment in the case of the absence of an external field (B_e = 0) takes on the form

$$\frac{d\bar{\beta}}{d\bar{t}} + 2\bar{\beta}\left(\frac{1+\kappa\bar{t}}{2} - \frac{\kappa}{1+\kappa\bar{t}}\right) = 0\,.$$

If by the moment of leaving the region of action the magnetic-field induction in the CJ segment had the value of B_0, this equation should be integrated under the initial condition $\bar{\beta}(0)$ = 2. As a result, for determining the magnetic-field induction on the jet-segment axis, we obtain the relation

$$\bar{B}_{\text{rod}} = (1+\kappa\bar{t})\,\exp\left[-\bar{t}\left(1+\frac{\kappa}{2}\bar{t}\right)\right]. \tag{8}$$

The parameter κ entering into the dependence obtained, as well as the diffusion-time constant τ, which is the time scale ($\bar{t} = t/\tau$), are calculated on the basis of relations (4) and (6) by the parameters of the CJ segment (radius R_{in} and strain rate $\dot{\varepsilon}_{z\text{in}}$) it has at the moment of leaving the magnetic field.

Figure 6 illustrates the change in the magnetic induction on the axis of the jet segment leaving the region of action at various values of the parameter κ. Depending on the value of κ, the magnetic field diffuses from the CJ segment monotonically or after the preamplification stage that is due to the stretching factor. As the analysis of relation (8) shows, amplification of the field occurs provided the condition $\kappa > 1$ is fulfilled. In so doing, the magnetic-field induction in the jet upon its leaving the region of action increases and at the time $t_{\max} = \tau(\sqrt{\kappa} - 1)/\kappa$ reaches its maximum and then drops to zero.

$$B_{\text{rod max}} = B_0\sqrt{\kappa}\,\exp\left(-\frac{\kappa-1}{2\kappa}\right),$$

Effect of the Magnetic Field on the Stressed State in the CJ. The result of the interaction between the magnetic field and the induced currents arising in the process of its evolution in the CJ material is the appearance of radial ponderomotive forces with the space density $f_r = jB$ acting on the jet, which should influence the stress-strain state in the jet, thus affecting the development of plastic instability. In accordance with the model of deformation of a rod with lateral surface disturbances [11] based on the hypothesis of plane sections, the localization of strains in the regions of necks nucleating on the CJ is determined by the distribution of the axial forces acting in its cross sections. Let us consider the influence of the magnetic field on the value of these forces. To this end, we determine the axial stresses σ_z arising in the jet upon its deformation in the magnetic field.

According to our model of an incompressible, rigid-plastic, cylindrical rod stretching at the axial strain rate $\dot{\varepsilon}_z$, the radial, tangential, and axial components of the deviator of the stress tensor in the jet have the following values: $s_r = s_\theta = \sigma_Y/3$ and $s_z = 2\sigma_Y/3$, where σ_Y is the flow limit of the jet material [5]. The motion of the jet particles in the radial direction in each of its cross sections with regard for the action of electromagnetic forces occurs in accordance with the equation

$$\rho \frac{dv_r}{dt} = \frac{\partial \sigma}{\partial r} + jB \,, \tag{9}$$

where v_r is the radial component of the jet-particle velocity, ρ is the density of the jet material, and σ is the mean stress in the jet material. Assuming the outer surface of the jet ($r = R$) to be free from the action of the stresses, $\sigma_r(R) = s_r + \sigma(R) = 0$, for the mean stress at the section boundary we will have a value of $\sigma(R) = \sigma_Y/3$. Integrating Eq. (9) over the cross-section radius with regard for the linearity of the radial velocity distribution $v_r = -\dot{\varepsilon}_z r/2$ following from the assumed condition of incompressibility of the jet material, for describing the mean stress in the jet, we obtain the equation

$$\sigma(r) = \frac{\sigma_Y}{3} - \frac{\rho}{8}\left(\dot{\varepsilon}_z^{\,2} - 2\frac{d\dot{\varepsilon}_z}{dt}\right)(R^2 - r^2) + \int_r^R jBdr \,.$$

Defining then the axial stresses as $\sigma_z = s_z + \sigma$ and averaging them over the cross-section area of the jet

$$\sigma_{\mathrm{zm}} = \frac{2}{R^2}\int_0^R \sigma_z r dr \,,$$

for calculating the mean axial stress acting in the jet, we finally arrive at the dependence

$$\sigma_{\mathrm{zm}} = \sigma_Y - \frac{\rho}{16}\left(\dot{\varepsilon}_z^{\,2} - 2\frac{d\dot{\varepsilon}_z}{dt}\right)R^2 + \frac{2}{R^2}\int_0^R\left(\int_r^R jBdr\right) r dr \,.$$

The last term in this dependence, which is absent upon free deformation of the jet and is due to the action of the electromagnetic forces, will be hereinafter referred to as the magnetic component of the mean axial stress:

$$\sigma_{\mathrm{zm}}^{\mathrm{mag}} = \frac{2}{R^2}\int_0^R\left(\int_r^R jBdr\right) r dr \,.$$

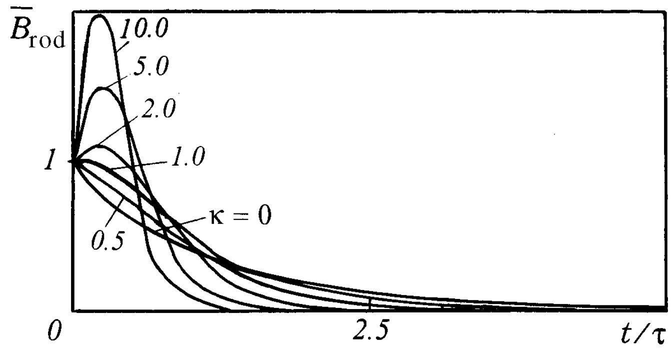


Fig. 6. Change in the magnetic field on the axis of a uniformly stretching element of a cumulative jet as it leaves the region of action.

Taking into account that $j = -(\partial B/\partial r)/\mu_0$, we can rewrite the expression for the magnetic component in a somewhat different form:

$$\sigma_{zm}^{mag} = \frac{2}{R^2}\int_0^R \frac{B^2}{2\mu_0} r dr - \frac{B_e^2}{2\mu_0} . \tag{10}$$

As is seen from the latter relation, the magnetic correction to the axial forces in the jet depends on the radial distribution of magnetic induction in its material. In one limiting case with the concentration of induced currents near the jet surface, the magnetic field inside the jet is homogeneous. Another limiting case was used in the simplified calculation of electromagnetic processes in the jet and corresponds to deep penetration of induced currents into its material with the realization of their linear distribution over the jet radius. In this case, the radial magnetic induction distribution has a parabolic form (2), and with the use of the induction value on the jet segment axis $B_{rod} = B_e + \mu_0\beta R^2/2$ we represent it in the following form:

$$B (r, t) = B_{rod} - (B_{rod} - B_e) \frac{r^2}{R^2} . \tag{11}$$

As analysis of relation (10) shows, for the considered conditions for change in the external field at one and the same magnetic induction B_{rod} on the jet axis in the first limiting case the absolute value of the magnetic force is maximum and in the second case it is minimum. Obviously, in a real situation the magnetic-induction distribution in the jet must have some intermediate character between the above limiting cases. Taking this into account, it appears to be justifiable to orient oneself to the mean between its values corresponding to these limiting cases. Then, calculating these values on the basis of relation (10) with the use of (11), we obtain

$$\sigma_{zm}^{mag} = \frac{B_e^2}{12\mu_0}\left(4\frac{B_{rod}^2}{B_e^2} + \frac{B_{rod}}{B_e} - 5\right). \tag{12}$$

If the external magnetic-field induction is larger than the induction of the field inside the jet ($B_{rod}/B_e < 1$), which takes place at the stage of penetration of the field into the jet when it enters the region of action, the mean axial stress acting in the jet decreases ($\sigma_{zm}^{mag} < 0$). On the contrary, if, owing to the generation caused by the stretching factor, the magnetic-field intensity in the jet exceeds the external field intensity ($B_{rod}/B_e > 1$), the axial forces increase ($\sigma_{zm}^{mag} > 0$).

When plastic instability develops on a CJ, the generation of a magnetic field inside the jet must proceed most intensively at the places of neck initiation where localization of axial strains takes place. The in-

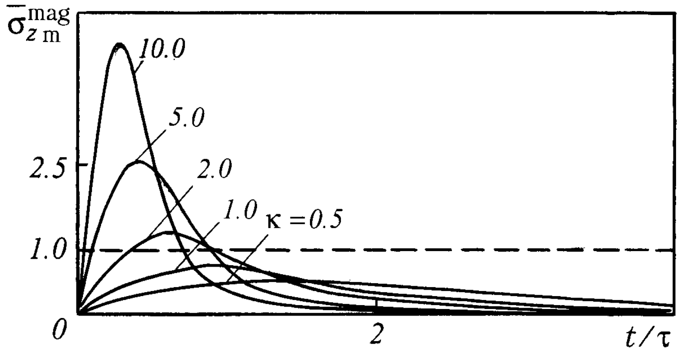


Fig. 7. Change in the magnetic component of the mean axial stress in a cumulative jet segment uniformly stretching with the magnetic field that has penetrated into its material.

crease in the axial forces in the necking regions accompanying this generation can, in principle, stabilize the process of CJ stretching at the necked stage.

To estimate the magnetic-field parameters that enable us to count upon a slowdown of the development of plastic instability on a CJ, we determine how significantly the mean axial stress has increased in the process of field generation in the stretching jet. Based, as before, on the model of a uniformly stretching rod, in order to determine the magnetic-field induction in the jet, we make use of the solution of the differential equation (5) under the initial condition $\beta(0) = 0$. Obviously, such a formulation of the problem corresponds to the deformation in the constant magnetic field with induction $B_e = B_0$ of the CJ element in whose material at zero time there exists a field of the same intensity as the external field. Equation (5) was integrated numerically, since the expression for calculating the magnetic induction on the CJ segment axis

$$\bar{B}_{rod} = 1 + \kappa\,(1 + \kappa\bar{t})\,\exp\left[-\bar{t}\left(1 + \frac{\kappa}{2}\bar{t}\right)\right]\int_0^{\bar{t}} \frac{\exp\left[\bar{t}\left(1 + \frac{\kappa}{2}\bar{t}\right)\right]}{(1 + \kappa\bar{t})^2}\,d\bar{t}$$

obtained on its basis includes an integral inexpressible in the final analytical form.

We relate the magnetic component of the axial stress determined on the basis of Eq. (12) to the magnetic pressure value $p_{mag} = 0.5B_0^2/\mu_0$ corresponding to the external field intensity. The change in the thus-obtained dimensionless magnetic stress

$$\bar{\sigma}_{zm}^{mag} = \frac{\sigma_{zm}^{mag}}{p_{mag}} = \frac{1}{6}\,(4\bar{B}_{rod}^2 + \bar{B}_{rod} - 5)$$

in the jet stretching with the magnetic field that has penetrated into its material is illustrated in Fig. 7 for various values of the parameter κ.

Upon free deformation, the mean axial stress in the CJ segments by the onset of the necked stage practically coincides with the flow limit of the jet material [5]. Taking this into account, in the first approximation one can obviously count on stabilization of CJ stretchings in the magnetic field on condition that the magnetic correction to the axial stress σ_{zm}^{mag} is comparable to the flow limit σ_Y. This condition is fulfilled if the intensity of the acting magnetic field satisfies the relation

$$B_0 \approx \sqrt{\frac{2\mu_0\sigma_Y}{\bar{\sigma}_{zm}^{mag}}}\,, \qquad (13)$$

where its maximum value determined as a function of the parameter κ by the plots can be taken as the characteristic value of the dimensionless magnetic stress $\bar{\sigma}_{zm}^{mag}$ (see Fig. 7). As estimates show, for the middle and rear segments of copper CJs of real shaped charges, by the onset of the necked stage of stretching the parameter κ assumes values from 0.1 to 1.0. The range of changes in the characteristic values of the quantity $\bar{\sigma}_{zm}^{mag}$ (Fig. 7) is 0.1–0.5. Taking these estimates into account, from relation (13) at $\sigma_Y \approx 0.2$ GPa (for copper CJs) it can be established that a noticeable manifestation of the stabilizing effect should be observed upon CJ deformation in a magnetic field with an induction on the order of several dozens of tesla.

NOTATION

r, z, radial and axial coordinates; t, time; $\bar{t}$, dimensionless time; B, magnetic-field induction; B_0, constant magnetic-field induction; B_e, external magnetic-field induction; B_{rod} and $\bar{B}_{rod}$, magnetic-field induction on the rod axis and its dimensionless value; $B_{rod\,max}$ and $\bar{B}_{rod\,max}$, maximum value of magnetic field induction on the rod axis and its dimensionless value; j, space density of current; β and $\bar{\beta}$, coefficient in the radial distribution of the space density of current and its dimensionless value; ρ, medium density; $\dot{\varepsilon}_z$, axial velocity of deformation; $\dot{\varepsilon}_{zin}$ and R_{in}, initial values of the axial velocity of deformation and rod radius; v_r and v_z, radial and axial components of the velocity vector; f_r, space density of the radial electromagnetic force; σ_r and σ_z, radial and axial components of the stress tensor; s_r, s_θ, and s_z, radial, tangential, and axial components of the stress deviator; σ_{zm}^{mag} and $\bar{\sigma}_{zm}$, magnetic component of the mean axial stress and its dimensionless value; P_{mag}, magnetic pressure; κ, dimensionless parameter characterizing the relation between the rates of diffusion and magnetic-field generation; t_e and $\bar{t}_e$, time of magnetic-field penetration into the rod material and its dimensionless value; κ_0, τ_0, and t_{e0}, values of the dimensionless parameter, time constant of diffusion, and time of magnetic-field penetration for CJ elements at the time of their formation; t_{max} and $\bar{t}_{max}$, time of reaching the maximum value of magnetic-field induction and its dimensionless value; d_z, diameter of shaped charge; h_0, height of cumulative coating; k_s, geometrical similarity factor; n, stretch ratio of the CJ segment; l_0 and l, initial and current length of the CJ segment; I_0 and I_1, zero- and first-order Bessel functions of the first kind, respectively; ν_k, positive roots of the equation $J_0(\nu) = 0$.

REFERENCES

1. W. P. Walters and J. A. Zucas, *Fundamentals of Shaped Charges*, New York (1989).
2. P. C. Chou and J. Carleone, *J. Appl. Phys.*, **48**, No. 10, 4187–4194 (1977).
3. L. A. Romero, *J. Appl. Phys.*, **65**, No. 8, 3006–3016 (1989).
4. A. V. Babkin, S. V. Ladov, V. M. Marinin, and S. V. Fedorov, *Prikl. Mekh. Tekh. Fiz.*, **40**, No. 4, 25–35 (1999).
5. A. V. Babkin, S. V. Ladov, V. M. Marinin, and S. V. Fedorov, *Prikl. Mekh. Tekh. Fiz.*, **38**, No. 2, 3–9 (1997).
6. S. V. Fedorov, A. V. Babkin, and D. S. Botkin, in: *Abstracts of the IVth Int. Conf. "Lavrent'ev Readings on Mathematics, Mechanics, and Physics,"* Kazan–Novosibirsk, July 3–7, 1995 [in Russian], Kazan–Novosibirsk (1995), p. 125.
7. A. V. Babkin, V. M. Marinin, and S. V. Fedorov, *Oboronnaya Tekhnika*, No. 9, 40–46 (1993).
8. L. D. Landau and E. M. Lifshits, *Electrodynamics of Solid Media* [in Russian], Moscow (1982).
9. A. N. Tikhonov and A. A. Samarskii, *Equations of Mathematical Physics* [in Russian], Moscow (1966).
10. V. M. Marinin, A. V. Babkin, and V. I. Kolpakov, *Oboronnaya Tekhnika*, No. 4, 34–39 (1995).
11. S. V. Fedorov, A. V. Babkin, and S. V. Ladov, *Oboronnaya Tekhnika*, Nos. 1–2, 49–56 (1998).